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Direct numerical simulation of stationary particles in homogeneous turbulence decay: Application of the k–ε model

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ABSTRACT

This study focuses on understanding how the presence of particles, in homogeneous turbulence decay, affects the dissipation of dissipation coefficient within the volume averaged dissipation transport equation. In developing this equation, the coefficient for dissipation of dissipation was assumed to be the sum of the single phase coefficient and an additional coefficient that is related to the effects of the dispersed phase. Direct numerical simulation was used to isolate the effect of stationary particles in homogeneous turbulent decay at low Reynolds numbers (Re_L = 3.3 and 12.5). The particles were positioned at each grid point and modeled as point forces and a comparison was made between a 64^3 and 128^3 domain. The results show that the dissipation of dissipation coefficient correlates well with a dimensionless parameter called the momentum coupling factor.

1. Introduction

In general, the effect of particles on carrier phase turbulent flow is inherently complicated. This complexity arises from the no-slip condition at the particle surfaces. The experiments of Tsuji et al. (1984), Sheen et al. (1993), Kulick et al. (1993), Paris and Eaton (2001) show various trends of turbulent kinetic energy (TKE) based on the particle diameter and mass loading. These trends suggest that particle size and loading have an effect on TKE. The work of Gore and Crowe (1989) suggests that the ratio of particle diameter to the length scale of the most energetic eddies in the flow (D/L) could be used to separate augmentation of turbulence intensity from attenuation when compared with the un-laden cases. They found that the turbulence intensity increased for $D/L > 0.1$ and decreased for $D/L < 0.1$. Hestroni (1989) suggests that a particle Reynolds number greater than 110 causes vortex shedding, which enhances the turbulence. The work of Kenning and Crowe (1997) showed that the inter-particle length scale may be an important parameter to consider in particle induced turbulent flows.

Based on experimental data (Tsuji et al., 1984; Kulick et al., 1993; Sheen et al., 1993) it is evident that additional surfaces within the flow alter the turbulent intensity relative to single phase flows. It is also apparent from the literature, as well as dimensional analysis, that there are four key non-dimensional variables: (1) fluid Reynolds number, (2) particle Reynolds number, (3) particle concentration, and (4) Stokes number. Therefore, it is hypothesized that the change in turbulent length and time scales due to the presence of particles must be influenced by one or more of these key non-dimensional variables, and a viable model of turbulence in a particle laden flow should include these fundamental parameters.

Several models have been developed to analyze and predict the levels of carrier-phase turbulence in particle laden flows. These models are typically categorized according to coupling (Elghobashi, 1994). Many of the models developed in the past have been based on either one-way or two-way coupling, few on four-way coupling.

Elghobashi and Abou-Arab (1983) developed a two-way coupled k–ε model for particle laden flows. The model assumes that the particles behave as a continuous medium. Their approach to obtaining the TKE and dissipation rate equations involved temporal averaging the volume averaged equations. The validity of this approach is open to argument, and the application may be limited to dusty gas conditions.

Following the work of Elghobashi and Abou-Arab (1983), Chen and Wood (1985) added a term to the single phase momentum equation to represent the force per unit volume applied to the fluid due to the presence of particles. The momentum equation assumes the form

$$\rho\frac{\partial u_i}{\partial t} + \rho u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j^2} + \frac{\rho_p}{t_p}f(v_i - u_i) \quad (1)$$

where ρ_p is the particle bulk density, t_p is the particle response time, f is the drag factor, and u_i and v_i are the fluid and particle velocity defined at a point in the flow. By applying the Reynolds averaging

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procedures used for single phase flow, Chen and Wood (1985) proposed the following equation for turbulence kinetic energy of the fluid phase:

$$\frac{Dk_t}{Dt} = -\overline{u_i'u_j'}\frac{\partial \overline{u_i}}{\partial x_j} - \frac{k_t}{3}\frac{\partial \overline{u_i}}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\frac{\nu_T}{\sigma_\epsilon}\frac{\partial k_t}{\partial x_i}\right] - \epsilon_t + \frac{\bar{\rho}_p}{\rho t_p}\left(\overline{u_i'v_i'} - \overline{u_i'u_i'}\right) + \frac{1}{\rho t_p}\left(\left(\overline{\rho_p' u_i' v_i'} - \overline{\rho_p' u_i' u_i'}\right)\right) + \frac{1}{\rho t_p}\left(\overline{v_i} - \overline{u_i}\right)\overline{\rho_p' u_i'} \tag{2}$$

where k_t is the time averaged turbulent kinetic energy, ε_t is the time averaged dissipation, ν_t is the turbulent viscosity, and σ_ε is the effective Schmidt number for turbulent diffusion. Chen and Wood (1985) also proposed the dissipation transport equation for the continuous phase to be of the form

$$\frac{D\epsilon_t}{Dt} = -C_{\epsilon 1}\overline{u_i'u_j'}\frac{\partial \overline{u_i}}{\partial x_j}\frac{\epsilon_t}{k_t} - C_{\epsilon 2}\frac{\epsilon_t^2}{k_t} - C_{\epsilon 3}\frac{k_t}{3}\frac{\partial \overline{u_i}}{\partial x_i}\frac{\epsilon_t}{k_t} + \frac{\partial}{\partial x_i}\left[\frac{\nu_T}{\sigma_\epsilon}\frac{\partial \epsilon_t}{\partial x_i}\right] + \frac{2\overline{\rho_p}}{\rho t_p}\left[\nu\overline{\frac{\partial u_i'}{\partial x_j}\left(\frac{\partial v_i'}{\partial x_j} - \frac{\partial u_i'}{\partial x_j}\right)}\right] \tag{3}$$

where $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, and $C_{\varepsilon 3}$ are fit coefficients.

The particle momentum equation, as shown in Eq. (1), has been used extensively in the development of models for turbulence modulation in multiphase flows (Yan et al., 2007; Nasr and Ahmadi, 2007; Mohanarangam and Tu, 2007; Vermorel et al., 2003; Kulick et al., 1993, to name a few). In order to treat any flowing medium as a continuum and justify the use of differential operators, the size (diameter) of the limiting volume (White, 1994) must be much less than the characteristic dimension of the flow. Eq. (1) is based on the assumption that the particulate phase can be regarded as a continuum and, moreover, the size of the limiting volume is comparable to that of the carrier phase. However, consider the diameter of the limiting volume of air (~1 μm) at standard conditions (White, 1994). The corresponding limiting diameter for a cloud of 10 μm particles with a concentration of 10 is on the order of 10^4; four orders of magnitude larger than the carrier phase. Thus the limiting volume used to develop Eq. (1) must be the largest of the two-phases, so it can only be that for the particulate phase. The velocity of the carrier phase will vary throughout the limiting volume so the carrier phase velocity in Eq. (1) can at best be regarded as a velocity averaged over the volume. It does not reflect the velocity at a point in the carrier phase so turbulence models developed using single-phase Reynolds averaging procedures are open to question.

Volume averaging and ensemble averaging provide a scheme to include the effects of the dispersed phase without the necessity of including the details of the surface interaction. Crowe et al. (1998) and Slattery (1972) provide a detailed description of the volume average concept. Aside from temporal averaging, another way of defining turbulence is by the velocity deviation from the volume averaged velocity at an instant in time (Crowe and Gillandt, 1998), such as

$$u_i = \langle u_i \rangle + \delta u_i \tag{4}$$

where u_i is the instantaneous velocity, $\langle u_i \rangle$ is the phase volume averaged velocity, and δu_i is the velocity deviation as illustrated in Fig. 1. One advantage of volume averaging is that the effects of particle surfaces appear fundamentally in the governing equations. This approach is not based on treating the particle phase as a continuum.

Crowe and Gillandt (1998) derived a turbulent kinetic energy equation for two-way coupled particle laden flows. They defined the volume average turbulent kinetic energy as

$$k = \frac{1}{V_c}\int_{V_c}\frac{\delta u_i \delta u_i}{2}\,dV = \left\langle \frac{\delta u_i \delta u_i}{2} \right\rangle \tag{5}$$

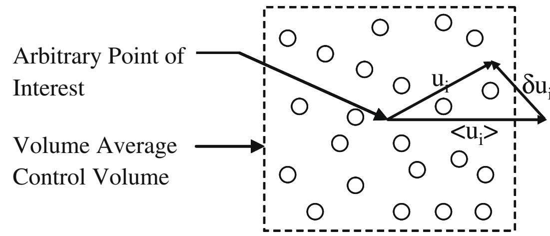

Fig. 1. Concept of modeling turbulence within a volume average setting.

and the volume averaged turbulent dissipation as

$$\epsilon = \frac{\nu}{V_c}\int_{V_c}\frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j}\,dV = \nu\left\langle \frac{\partial \delta u_i}{\partial x_j}\frac{\partial \delta u_i}{\partial x_j} \right\rangle \tag{6}$$

Their transport equation for turbulent kinetic energy is found to be

$$\alpha_c\frac{Dk}{Dt} = -\alpha_c\langle \delta u_i \delta u_j \rangle\frac{\partial \langle u_i \rangle}{\partial x_j} - \frac{\partial}{\partial x_j}\left[\alpha_c\langle \delta u_j k\rangle\right] - \frac{1}{\rho}\frac{\partial}{\partial x_i}\left(\alpha_c\langle \delta P \delta u_i \rangle\right) + 3\pi\upsilon\sum_n D_n f_n[u_i - v_i]_n[\langle u_i \rangle - v_i]_n + \alpha_c\left\langle \frac{\partial}{\partial x_j}\left(\nu\frac{\partial k}{\partial x_j}\right)\right\rangle - \alpha_c\epsilon \tag{7}$$

In comparison to the time averaged models discussed above, the above TKE model treats the particles individually rather than as a continuum. In order to close the volume average equation set for particle laden flows, a volume averaged dissipation transport equation is needed.

The dissipation equation is difficult to understand for single phase flows; the complexities involved in coupling the continuous phase dissipation to the dispersed phase within multiphase flows can be even more challenging. As an interim solution, many researchers have added a term to the single phase dissipation equation to account for the dispersed phase effects.

Lain et al. (1999) added an additional term to the dissipation equation that accounted for the dissipation due to the presence of particles. This additional term is proportional to the energy generation and redistribution terms proposed by Crowe and Gillandt (1998). The coefficient for this additional term was determined to be a constant of 1.1. Squires and Eaton (1992) claim that the coefficient is not a universal constant but potentially a function of bulk density and diameter of the particle.

Zhang and Reese (2003) proposed a dissipation transport equation that contained an additional term due to the presence of particles and is modeled after Crowe and Gillandt (1998) generation and redistribution terms. In their model, they included the interparticle length scale in the form of a reduced turbulent length scale. Zhang and Reese showed good agreement between the model and the experimental data of Tsuji et al. (1984).

Recently, Schwarzkopf et al. (in press) derived a dissipation transport model using volume averaged techniques to support the work of Crowe and Gillandt (1998). The model is of the form

$$\alpha_c\frac{D\epsilon}{Dt} = C_{\epsilon 1}'\alpha_c\frac{P_\epsilon \epsilon}{k} - C_{\epsilon 2}'\alpha_c\frac{\epsilon^2}{k} + C_{\epsilon 3}\frac{\upsilon^2}{V}\sum_n f_n\frac{|u_i - v_i|_n^2}{D_n} + \frac{\partial}{\partial x_i}\left[\alpha_c\left(\nu + \frac{\upsilon_T}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_i}\right] \tag{8}$$

where α_c is the void fraction, P_ε is the production of dissipation, $C_{\epsilon 1}'$ and $C_{\epsilon 2}'$ are the combined single and dispersed phase production and dissipation of dissipation coefficients respectively, $C_{\varepsilon 3}$ is the coefficient for production of dissipation due to particles, V is the mixture volume, ν is the kinematic viscosity, f is the drag factor, u_i and v_i are the instantaneous fluid and particle velocities at the particle location, respectively, D is the particle diameter, n is the particle number within the mixture volume, ν_T is the turbulent viscosity, σ_ε is the effective Schmidt number for turbulent diffusion, and k and ε are the volume averaged turbulent kinetic energy and dissipation (defined in Eqs. 5 and 6). The additional term is a production of dissipation due to the presence of particles. The form of this term was arrived at by integrating the deviation velocity gradients around a single particle within a Stokes drag regime and then applying a summation to represent all the particles within a mixture volume. A constant was then replaced with a coefficient in order to apply the term to turbulent flows. The advantage of this model over previous models is the fact that the effects of particle surfaces within the finite volume can be individually accounted for. Because the surface effects are included by integrating properties around the particle surfaces, the model inherently includes the effects of inter-particle spacing. The ratio of the production of dissipation coefficient due to particles ($C_{\varepsilon 3}$) and the dissipation of dissipation coefficient ($C'_{\epsilon 2}$) was found by applying the above equation to the case of homogeneous turbulence generated by particles falling in a bath of fluid, and determined to be of the form (Schwarzkopf, 2008)

$$\frac{C_{\epsilon 3}}{C'_{\epsilon 2}} \approx C_{\epsilon p}(\mathrm{Re}_r)^m \tag{9}$$

where $C_{\varepsilon p}$ = 0.0587 and m = 1.4161. The coefficient for the production of dissipation due to mean velocity gradients is assumed to be of the form

$$C'_{\epsilon 1} = C_{\epsilon 1} + C_{\epsilon 1p} \tag{10}$$

where $C_{\varepsilon 1}$ is the well known single phase coefficient (1.44) and $C_{\varepsilon 1p}$ is the contribution of the dispersed phase. The coefficient for the dissipation of dissipation is assumed to be of the form

$$C'_{\epsilon 2} = C_{\epsilon 2} + C_{\epsilon 2p} \tag{11}$$

where $C_{\varepsilon 2}$ is the well known single phase coefficient (1.92) and $C_{\varepsilon 2p}$ is the contribution of the dispersed phase. It is required that these additional coefficients approach zero as the mixture approaches single phase; this ensures that the single phase k–ε model is retained when particles are not present.

In single phase flows, the dissipation of dissipation coefficient is typically calibrated for homogeneous, isotropic turbulence decay. However, for the case of particles in isotropic homogeneous turbulence, there are a limited number of data sets. Schreck and Kleis (1993) obtained measurements of neutrally buoyant plastic and heavy glass particles in homogeneous turbulent decay of water. Their studies show that attenuation of turbulent kinetic energy increased with increased particle loading. Geiss et al. (2004) showed similar results. The experimental data needed to calibrate the dissipation of dissipation coefficient ($C'_{\epsilon 2}$) are insufficient; however, direct numerical simulations (DNS) can be used to find the particle contribution to the dissipation of dissipation coefficient.

Direct numerical simulation has been used by several researchers to study the effects of particles on the modulation of turbulence within the carrier phase. Squires and Eaton (1990) modeled a large number of particles using direct numerical simulation. The turbulence was forced at low wave numbers and the governing equations were solved using pseudo-spectral methods. The Reynolds number (Re_L) was ~37 and the solution was shown to be grid independent by increasing the number of grid points from 32^3 to 64^3. The particles were modeled as point forces using the particle-source-in-cell (PSIC) method. Their results showed that the presence of particles (for a mass loading from 0.1 to 1) tend to decrease the turbulent kinetic energy and dissipation relative to un-laden flows. Ferrante and Elghobashi (2003) also used DNS techniques to study the effect of a large number of particles in the flow. They used second order finite differencing rather than pseudo-spectral methods on a 256^3 grid and tracked upwards of 8×10^7 particles. The results of their work shows that the presence of particles can decrease the turbulent kinetic energy (TKE), increase the TKE or provide no change in TKE relative to un-laden TKE. They claim that the Stokes number is an important parameter. Burton and Eaton (2005) used an overset grid technique to directly resolve the flow around a single particle in a turbulent field. Their simulations included 192^3 and 384^3 grid sizes. By comparing laden simulations to un-laden, they found that within 1.5 particle diameters from the surface the turbulence in the carrier phase was altered and beyond 5 particle diameters the carrier phase turbulence was not affected.

The purpose of this study is to determine how the presence of the particles affects the dissipation of dissipation coefficient in the volume averaged dissipation model and to identify the key parameters on which it depends. In this case the particle Reynolds number is small in order to suppress the production of dissipation by particles. Yet it is still likely that the dissipation coefficient will be influenced by a low particle Reynolds number. The current study represents an initial step to identify these key fundamental parameters. The inclusion of particle Reynolds number effects will be the subject of a continuing study.

2. Simulation

A spectral based code was used to simulate the scales of an isotropic homogeneous turbulent decay. The benefit is that no closure model is needed as the length and time scales within the flow are directly resolved. Orzag and Patterson (1972), Rogallo (1981) developed and used this method to study isotropic, homogeneous turbulence.

In a DNS environment, the Navier–Stokes equations are non-dimensionalized and solved for in Fourier space to maintain accuracy in determining the spatial gradients. The physical velocity is expressed using a Fourier series of the form

$$u_i(x_j, t) = \sum_k e^{ik_j x_j} \hat{u}_i(k_j, t) \tag{12}$$

and the orthogonality condition is applied to solve for the velocity coefficients. Within the Fourier domain, a point force is applied to distinct grid points to simulate a stationary particle. For incompressible flow, the non-dimensional Navier–Stokes equations with a point force representing a stationary particle at each grid point in the domain are

$$\frac{\partial \tilde{u}_i}{\partial \tilde{t}} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial \tilde{x}_j} = -\frac{\partial \tilde{P}}{\partial \tilde{x}_j} + \mathrm{Re}_L^{-1} \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_j^2} - \frac{3\pi \widetilde{D}}{\Delta \widetilde{V}} \mathrm{Re}_L^{-1} f \tilde{u}_i \tag{13}$$

where the equation was non-dimensionalized by an arbitrary fluid velocity (U) and the domain length scale (L). The tilde represents the non-dimensional form of each term, where $\tilde{t}$ is the non-dimensional time, $\widetilde{D}$ is the non-dimensional particle diameter, $\Delta\widetilde{V}$ is the non-dimensional volume of the computational cell, f is the drag factor, and Re_L is the Reynolds number. The energy spectrum used to initialize the domain was of the form

$$E(k) = \gamma \left(\frac{k/c}{1 + (k/c)^2} \right)^4 \tag{14}$$

where γ and c are constants (McMurtry, 1987; de Bruyn Kops and Riley, 1998). To ensure that the smallest scales are being modeled, the grid was set to be smaller than the Kolmogorov length scale ($\eta k_{\max} > 1$). To avoid aliasing effects, the 2/3 rule was applied to determine the maximum wave number based on the number of grid points chosen ($k_{\max} = N/3$) and spherical truncation was applied to the domain. All spatial derivatives were computed using

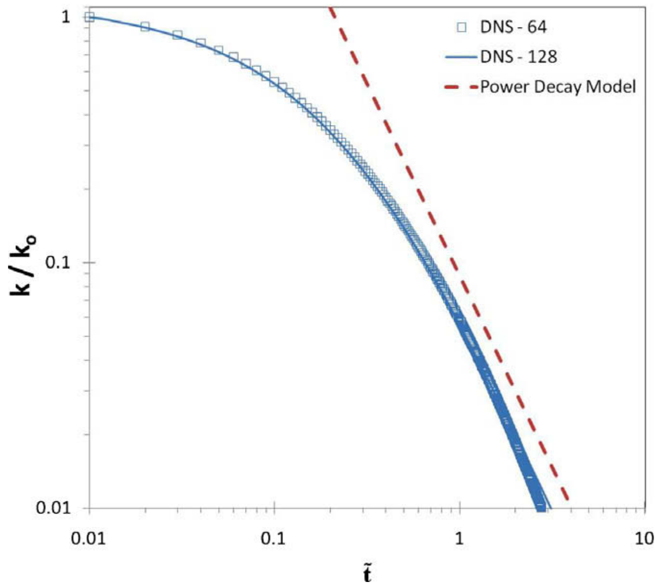

Fig. 2. Comparison of DNS results with the power decay model (Pope, 2000) for $n = 1.56$ over the non-dimensional time ($\tilde{t}$) for $Re_L = 12.5$: the decay curve is offset intentionally for clarity.

pseudo-spectral methods. The length of the domain was set at 2π and the boundary conditions were periodic. The time stepping routine was a second-order Adam–Bashforth algorithm. Additional details on the development of the code can be found in McMurtry, 1987. Since the particles are stationary, the force on the fluid due to the particles can be solved for in Fourier space by treating it like an increased viscosity (Schwarzkopf, 2008).

To validate the code, the non-dimensional viscosity was set to 0.08 ($Re_L = 12.5$) and the results of a 64^3 domain were compared to 128^3 (shown in Fig. 2). In addition, the decay rate predicted by the DNS model was compared to the decay power law $[k/k_o = (t/t_o)^{-n}]$ (Pope, 2000). For the case of $Re_L = 12.5$ and an exponent of $n = 1.56$, the slope of the decay model closely matches the slope of the DNS results (Fig. 2). The literature shows values for the decay exponent in the range of $1.15 < n < 1.45$ (Pope, 2000).

For the case of stationary particles of uniform size placed at every grid point within a homogeneous turbulent decay environment, the non-dimensional form of the dissipation transport model, Eq. (8), is

$$\alpha_c \frac{\partial \tilde{\epsilon}}{\partial \tilde{t}} = -C'_{\epsilon 2} \alpha_c \frac{\tilde{\epsilon}^2}{\tilde{k}} + C_{\epsilon 3} \frac{\tilde{\upsilon}^2}{\tilde{V}} f N_p \frac{2\tilde{k}}{\tilde{D}} \tag{15}$$

where N_p is the number of particles. From the DNS results, the volume averaged non-dimensional dissipation and kinetic energy can be obtained. In this study, the results of the DNS simulation are used to provide values for all the variables in Eq. (15) leaving only the coefficients as unknown. The time rate of change of the volume averaged non-dimensional dissipation is computed using a second-order finite difference scheme. The ratio of coefficients $(C_{\epsilon 3}/C'_{\epsilon 2})$ is shown in Eq. (9); the remaining coefficient $(C'_{\epsilon 2})$ in Eq. (15) can then be solved for.

3. Results and discussion

The assumed form of the dissipation of dissipation coefficient for particle laden flows is shown in Eq. (11). It is a requirement that $C_{\varepsilon 2p}$ approaches zero as the flow transitions to single phase. As the fluid transitions to single phase, the last term on the right hand side of Eq. (13) must go to zero; therefore either the slip velocity must go to zero, or the coefficient must go to zero. The obvious question is to understand if the particle dissipation of dissipation coefficient can be related to the non-dimensional particle momentum coupling factor, defined as

$$\Psi_p = 3\pi\tilde{\upsilon}\tilde{n}\tilde{D} \tag{16}$$

where $\tilde{n}$ is the dimensionless number of particles per unit volume, or number density (the above term can be found from $3\pi\tilde{D}(\Delta\tilde{V} \cdot Re_L)^{-1}$). A comparison between DNS simulations with and without stationary particles is used to assess the effect of the particles on the overall dissipation of dissipation coefficient.

The effect of the momentum coupling factor is shown in Fig. 3a and b. As the particle momentum coupling factor is increased, the particles rapidly attenuate the kinetic energy (Fig. 3a) and dissipation (Fig. 3b). The notion of decreased kinetic energy for increased particle loading is in agreement with the DNS work of Squires and Eaton (1990) and also the experimental work of Schreck and Kleis (1993). The trends for dissipation show that the slope is becoming more negative as the particle momentum coupling factor is increased; from Eq. (15) it can be seen that the additional production of dissipation term (due to particles) does not account for this case.

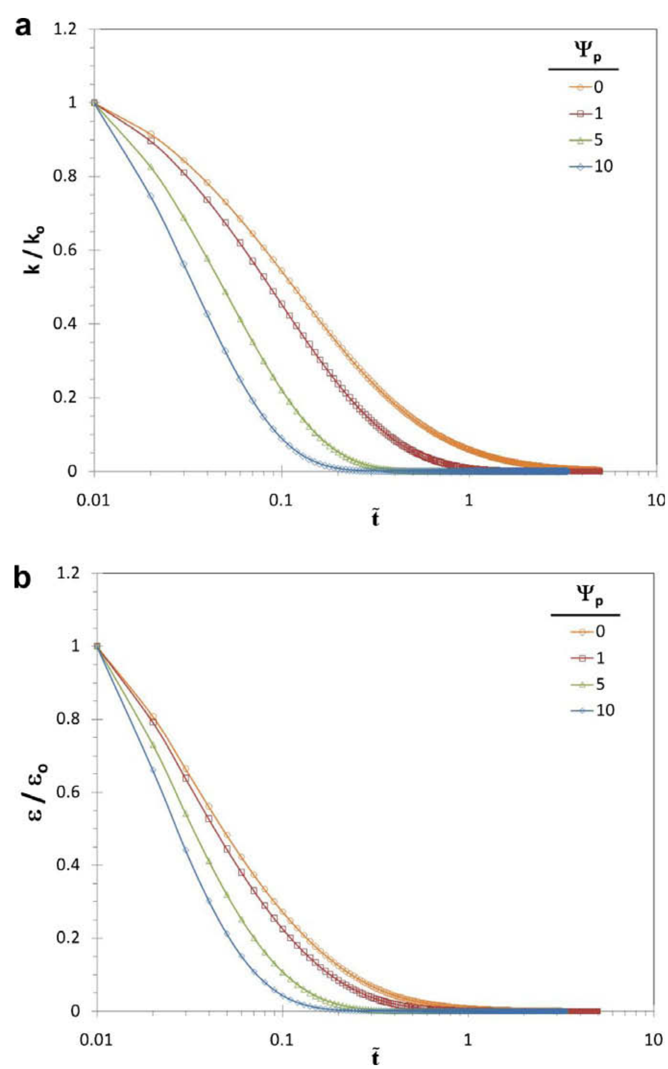

Fig. 3. DNS comparison of various particle loadings over the non-dimensional time ($\tilde{t}$) for $Re_L = 12.5$: (a) effect of particle loading on normalized TKE, (b) effect of particle loading on normalized dissipation (the TKE and dissipation are normalized by the initial value). The particle momentum coupling factor (Ψ_p) is described in Eq. (16).

Therefore, the dissipation of dissipation must balance with the production of dissipation due to particles and the time rate of change of dissipation due to turbulent decay. Based on the trends, it is hypothesized that the particle contribution to the dissipation of dissipation coefficient can be modeled as a function of the non-dimensional particle momentum coupling factor.

In order to validate this hypothesis, two different Reynolds numbers were used, $Re_L = 12.5$ and 3.3, additional turbulence and particle parameters are shown in Table 1. The Reynolds number of 12.5 is close to the turbulent decay region found in the literature ($n = 1.56$) and the Reynolds number of 3.3 is close to the final decay region ($n = 2.5$). An iterative method was used to determine the dissipation of dissipation coefficient – $C'_{\epsilon 2}$. The left hand side (LHS) of Eq. (15) was computed from DNS results for dissipation over a range of Taylor scale Reynolds numbers (4.5–0.01) that agreed with the decay law. The DNS results for k and ε were substituted into the right hand side (RHS) of Eq. (15) and $C'_{\epsilon 2}$ was iterated until the difference between the RHS and LHS of Eq. (15) was minimized. It was found that the additional production of dissipation term was negligible due to the low particle Reynolds number. Based on the results, it is clearly seen that the slopes are captured for several values of Ψ_p by merely increasing the coefficient – $C'_{\epsilon 2}$, shown in Fig. 4a and b.

With this method, the contribution to the coefficient of dissipation of dissipation due to the presence of particles can be isolated. The dissipation coefficient for the single phase ($C_{\epsilon 2}$) was determined from the DNS results with no particles. The effect of $C_{\epsilon 2p}$ is determined from Eq. (11) and plotted against the non-dimensional particle momentum factor that is re-scaled by the non-dimensional turbulence characteristics to rid the parameter of U

Table 1
Dimensionless flow parameters at the initial time associated with Reynolds numbers of 12.5 and 3.3 for a 64^3 domain.

Ψ_p	Re_L	Re_λ	Re_p	D/L	λ/L	η/L	α_d
1	12.5	4.8	1.5E−04	5.1E−06	0.2751	0.0636	1.82E−11
5	12.5	4.8	7.5E−04	2.55E−06	0.2751	0.0636	2.28E−09
10	12.5	4.8	1.5E−03	5.1E−05	0.2751	0.0636	1.82E−08
1	3.3	1.3	1.1E−05	1.35E−06	0.2751	0.1233	3.37E−13
5	3.3	1.3	5.3E−05	6.75E−06	0.2751	0.1233	4.22E−11
10	3.3	1.3	1.1E−04	1.35E−05	0.2751	0.1233	3.37E−10

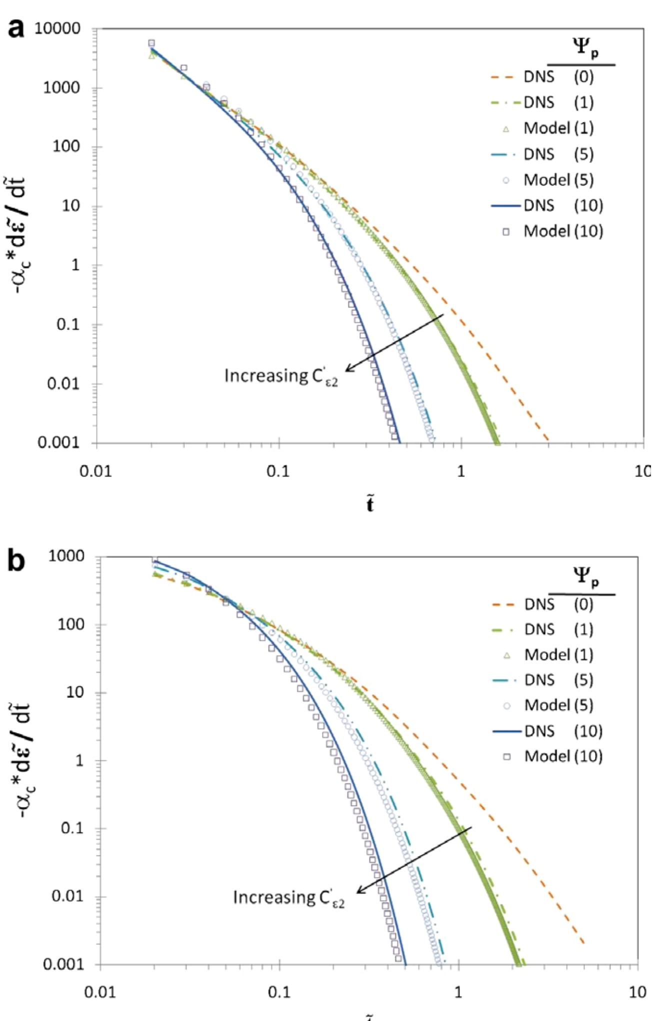

Fig. 4. Comparison of the dissipation model (Eq. (15)) with $C'_{\epsilon 2}$ to the DNS results over the non-dimensional time ($\tilde{t}$): (a) $Re_L = 12.5$, (b) $Re_L = 3.3$. These results show that the dissipation of dissipation coefficient, $C'_{\epsilon 2}$, can be increased to reflect the trends produced by increasing the particle momentum coupling parameter (Ψ_p), shown in Eq. (16).

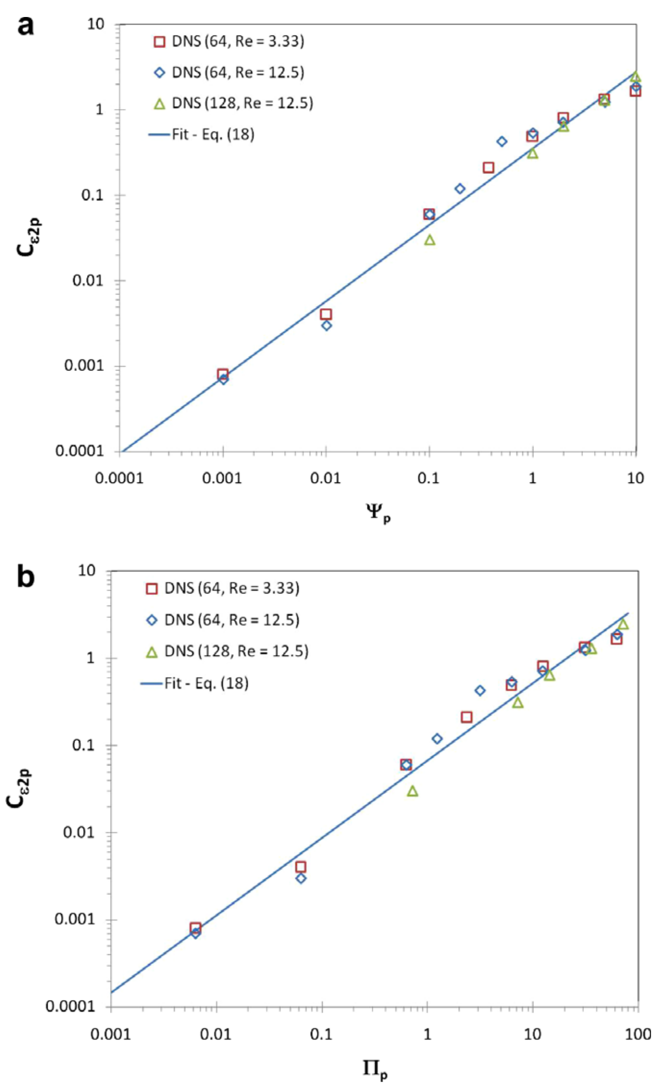

Fig. 5. The contribution of particles to the dissipation of dissipation coefficient ($C_{\varepsilon 2p}$) correlated with (a) the dimensionless particle momentum coupling factor (Ψ_p), and (b) the dimensionless particle-to-turbulence momentum coupling parameter (Π_p). The legend shows the number of grid points along each direction of the domain along with the Reynolds number. Eq. (18) is plotted for comparison.

and L. Thus a dimensionless particle-to-turbulence momentum coupling parameter is defined as

$$\Pi_p = 3\pi \upsilon n D \frac{\lambda_o}{\sqrt{k_o}} \quad (17)$$

where λ_o is the initial Taylor length scale and k_o is the initial turbulent kinetic energy.

The result shows that the effect of particles on the dissipation of dissipation coefficient ($C_{\varepsilon 2p}$) correlates well with Ψ_p and Π_p as shown in Fig. 5a and b, respectively. From the data shown in Fig. 5a, it is clear that for particle laden turbulent flows where Ψ_p is low, the effect of the particles on the dissipation of dissipation term is negligible, thus the single phase coefficient is adequate. However, for increased particle size or number density, the nature of the particles tends to increase the dissipation of dissipation (for the case of low fluid and particle Reynolds numbers). The results, as seen in Fig. 5b, also show that the effect of the particles on the dissipation of dissipation can also be scaled with the initial turbulence characteristics of the flow. For both cases, a correlation was developed using the least squares method and found to be of the form

$$C_{\epsilon 2p} = \xi(\Phi)^m \quad (18)$$

where $\xi = 0.3546$, $m = 0.893$ for $\Phi = \Psi_p$, and $\xi = 0.0675$, $m = 0.8868$ for $\Phi = \Pi_p$.

4. Conclusion

This paper focuses on determining the coefficient of dissipation of dissipation in the presence of isotropic homogeneous turbulence decay with stationary particles. The particles are represented by point forces and are placed at every grid point within a DNS domain. The study focused on two Reynolds numbers (12.5 and 3.3). The results show that the coefficient for the dissipation of dissipation due to the presence of particles can be correlated to the dimensionless particle-to-turbulence momentum coupling parameter (Π_p).

The simulations were carried out for the case of small particle Reynolds numbers. The results identify the importance of the momentum coupling factor on the coefficient for the dissipation of dissipation. Subsequent studies will include the contribution of the particle Reynolds number to the turbulence dissipation rate.

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